Cryptarithms

Fall 2016 ARML Power Contest

A *cryptarithm* is an arithmetic puzzle where digits are replaced by letters and the puzzle is to figure out which digit each letter stands for. One of the best known examples is a classic puzzle by one of the greatest puzzle creators of all time, Henry Dudeney.

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  S E N D
+ M O R E
---------
M O N E Y
```

Dudeney's puzzle is a special kind of cryptarithm called an *alphametic,* where the letters actually spell words that fit together in a sentence. The rules of cryptarithms are: 1) each letter stands for exactly one digit, and it is the same digit each time the letter is used, 2) no digit is represented by two or more different letters, and 3) the leading digit of a number cannot be zero.

Dudeney's puzzle can be solved as follows. (It is highly recommended that you get out some scratch paper and work along here!) First, a four-digit number is less than 10000, so `MONEY`, being the sum of two four-digit numbers, must be less than 20000. But since the leading digit `M` cannot be zero, it must be a one. Then the number `MORE` is less than 2000, so `MONEY` is less than 12000. Thus, the letter `O` must represent a 0 or a 1. But it cannot represent 1, since `M` already does, so it must represent 0.

Next, look at the hundreds column where it tells us that $\texttt{E} + 0 = \texttt{N}$. Since `E` and `N` must represent different digits, the only way that could happen is if there had been a carry from the addition in the tens column. So we know that `N` is one more than `E` (or that `E` is 9 and `N` is zero, but that is ruled out because we already know that `O` is 0). It also means that there is no carry from the hundreds place to the thousands place. Then we know that $\texttt{S} + 1 = 10$ so `S` is 9.

The table below summarizes what we know so far:

0	1	2	3	4	5	6	7	8	9
O	M								S

Additionally, we know that the pair `EN` represents 23, 34, 45, 56, 67, or 78. You will later be asked to finish this puzzle.

In problems that ask you to solve a cryptarithm, the answer should be given as in the table above. That is, if a problem asks you to *solve* a cryptarithm (as opposed to explaining something, etc.) then the answer is simply a table whose first row is the digits zero through nine (do not leave any out, even if they don't appear in the answer to the puzzle) and whose second row is the letter that represents the digit above it. You can earn partial credit for partial solutions. For example, you might earn one point out of three for the above table, even if you aren't able to figure out the values of the remaining letters.

In problems where a *restoration* is called for (problems 9–11) you will have a multiplication problem where the digits have been replaced by letters. Your answer should show the entire original multiplication (not just the factors and product!). In other words, show the entire long multiplication with the digits restored to their original positions.

One last thing to note. Many cryptarithms admit multiple solutions, while others have none at all. For example, YOU + ME = LOVE has no solution at all, while you can check that both

0	1	2	3	4	5	6	7	8	9
N	F	M	R	P	E	T	H	A	O

and

0	1	2	3	4	5	6	7	8	9
N	M	F	R	P	E	T	H	A	O

are solutions to MOTHER + FATHER = PARENT. (The roles of the M and F are switched.) In the puzzles for this contest, unless otherwise stated, *every puzzle will have a unique solution.*

Enjoy the puzzles!