Cryptarithms

Fall 2016 ARML Power Contest Problems

Problem 1. Let's explore why there are no solutions to the puzzle given in the reading:

```
  Y O U
+   M E
-------
L O V E.
```

[1] (a) Fill in the blanks with the smallest possible integers: Even if we were allowed to have different letters representing the same digit, the three-digit number YOU is at most _____ and the two-digit number ME is at most _____, so their four-digit sum LOVE is at most _____ . (Remember to put your answers on the answer page and not just on this question sheet!)

[1] (b) Continue filling in the blanks with the only digits that follow logically from your work in part (a): Thus, the only hope for an answer to this puzzle is if L represents the digit _____ and O represents _____ .

[2] (c) Now, remembering that we *cannot* have two different letters representing the same digit, explain why there cannot be any solution to this puzzle.

[2] **Problem 2.** Complete the solution to Dudeney's puzzle. Remember to give your answer in the proper format!

```
  S E N D
+ M O R E
---------
M O N E Y
```

[4] **Problem 3.** Solve the astronomical alphametic below.

```
  S A T U R N
+ U R A N U S
-------------
P L A N E T S
```

[3] **Problem 4.** A *doubly-true* alphametic occurs when the words spell out a true mathematical sentence. Try this one:

```
  F O R T Y
      T E N
+     T E N
---------
  S I X T Y.
```

[3] **Problem 5.** Why should addition have all the fun? The following multiplication problem is clearly not a doubly-true alphametic!

```
      T W O
*     T W O
---------
T H R E E
```

[3] **Problem 6.** There's nothing special about English, either. The following is doubly-true *auf Deutsch*.

```
    E I N S
    E I N S
    E I N S
+   E I N S
---------
    V I E R
```

[3] **Problem 7.** Get out your best factoring tricks to solve this one:

$$9(\mathtt{B\ I\ G\ T\ O\ P}) = 4(\mathtt{T\ O\ P\ B\ I\ G}).$$

[3] **Problem 8.** Here is another old classic, created by master puzzle creator Alan Wayne. Solve:

$$\mathtt{K\ I\ S\ S} = \sqrt{\mathtt{P\ A\ S\ S\ I\ O\ N}}.$$

[3] **Problem 9.** The following isn't strictly a cryptarithm, but it seems to be in the same spirit. In the following multiplication problem, each digit has been replaced with the same letter, `X`. And each digit in the original problem was a prime. There is only one solution. For your answer, restore the multiplication, that is, reproduce the entire multiplication, restoring as many digits as you can.

```
    X X X
*     X X
---------
  X X X X
X X X X
---------
X X X X X
```

[3] **Problem 10.** In the variant below (again, this isn't strictly a cryptarithm), each time the letter `E` occurs it should be replaced by an even digit (though not necessarily always the same digit), while each `O` is to be replaced by an odd digit (again, not necessarily the same odd digit each time) to make a correct multiplication. Can you restore all the digits?

```
    E E O
*     O O
---------
  E O E O
  E O O
---------
O O O O O
```

[4] **Problem 11.** This one is another odd/even restoration, only a little bit harder:

```
   O E E
 *   E E
 -------
 E O E E
 E O E
 -------
 O O E E.
```

[5] **Problem 12.** There's nothing special about base 10! Determine, with proof, all bases in which the following alphametic has at least one solution.

```
  K Y O T O
  K Y O T O
+ K Y O T O
-----------
  T O K Y O
```