2017 ARML Local Competition

Photocopying Instructions

Make one copy of the whole packet for each team. It contains:

1 copy of the Team Score Sheet
1 copy of the Team Round Answer Sheet
6 copies of the Team Round (2 pages)
1 copy of the Individual Round questions (for proctor)
6 copies of each Individual Round pair
1 copy of each Relay Round Sheet (6 pages for each round)
1 copy of the Relay Round Answer Sheets
(Cut out the 12 miniature answer sheets)
5 copies of the Tiebreaker Question (make more as needed)

Notes:
No calculators are allowed for any round.
Make sure copious scratch paper is available.
Thank you so much for coordinating ARML Local.

2017 ARML Local Competition

Team Score Sheet

Team Name: ________________________________

Team Round (4 pts per correct answer, 60 max.)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Tot

Individual Round (1 pt per correct answer, 60 max.)

Student Names	1	2	3	4	5	6	7	8	9	10	Tot
1.											
2.											
3.											
4.											
5.											
6.											
Totals											

Relay Round

(Round 1: 3x 2pts/1pt. Round 2: 2x 4pts/2pts, Round 3: 1x 6pts/3pts. 20 points max.)

Relay Round/Team	1/1	1/2	1/3	2/1	2/2	3	Total
Score							

Total (out of 140): __________

2017 ARML Local Competition

Team Round Answer Sheet

Team Name: ________________________________

Question Number	Answer
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

2017 ARML Local Team Round
(45 minutes)

T-1 A fair six-sided die has faces with values 0, 0, 1, 3, 6, and 10. Compute the smallest positive integer that *cannot* be the sum of four rolls of this die.

T-2 For a positive integer k, let z_k be the number of terminal zeroes of the product $1!2!\cdots k!$. For example, $z_6 = 2$ because $1!2!3!4!5!6! = 24883200$. Compute z_{100}.

T-3 Let $ARML$ be a square of side length 5. A point B on side $\overline{MR}$ and C on side $\overline{ML}$ are selected uniformly at random and independent of one another. Compute the expected area of triangle ABC.

T-4 The edges of regular hexagon $ABCDEF$ are made of mirrors. A laser is fired from A toward the interior of edge $\overline{CD}$, striking it at point G. The laser beam reflects off the interior of exactly one additional edge and returns to A. Compute $\tan(\angle DAG)$.

T-5 Let S be the set of lattice points $\{(x, y) : x, y \in \mathbb{Z}, 0 \leq x \leq 3, 0 \leq y \leq 4\}$. Compute the number of subsets of S of 4 lattice points that form the vertices of a square.

T-6 If A is an acute angle such that $\sin 15^\circ + \cos 15^\circ = \sqrt{2} \sin A$, compute $\cos A$.

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Step 2: Replace n with the product of its digits.
Step 3: If $n \leq 9$ return k and exit. If $n > 9$, increment k by 1 and return to Step 2.

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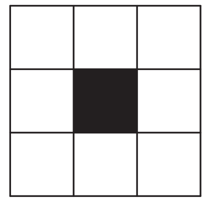
T-8 Three co-planar squares, $BAHT$, $CAIN$, and $BCGY$ have areas 16, 16, and 32, respectively. If the squares only intersect pairwise at the vertices A, B, and C, compute the area of the convex hexagon $THINGY$.

T-9 Compute the coefficient of x^8 in the expansion of $(x^2 + x + 1)^8$ after combining like terms.

T-10 Consider the following 3×3 grid with its center square removed, as shown.



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T-12 Compute the minimum value of ab such that $\log_2(a^4b^{-3}) = 3$ and $\log_2(a^4b^3) = 9$.

T-13 Compute

$$\sum_{n=3}^{10} \frac{20}{(n-2)(n+2)}$$

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T-14 The six-digit number 142857 has the property that moving the rightmost digit to the left of the number results in multiplying the number by five ($142857 \times 5 = 714285$). Compute the smallest six-digit number with the property that moving the rightmost digit to the left of the number results in multiplying the number by four.

T-15 Compute the number of distinct sequences $(x_1, x_2, \ldots, x_{14})$ with the following properties:

– $x_k \in \{1, 2, \ldots, 14\}$ for each $k = 1, 2, \ldots, 14$.
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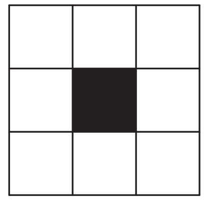

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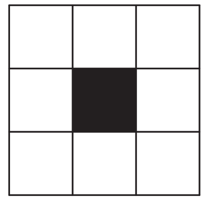

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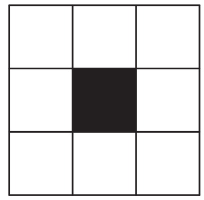

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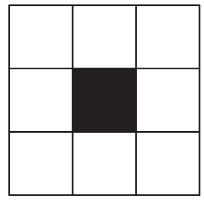

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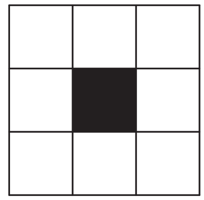

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Individual Round (10 minutes per pair)

I-1 A positive integer m is *stable* if $m = 2^n - n^2$ for some positive integer n. Compute the number of stable positive integers less than 2017.

I-2 Compute the area of the quadrilateral with vertices at $(1,1)$, $(4,7)$, $(5,3)$, and $(2,0)$.

I-3 The sum of the squares of five consecutive positive integers, the largest integer being n, is equal to the sum of the squares of the next four consecutive integers. Compute n.

I-4 The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the sum of its coefficients is equal to the sum of its roots and is also equal to the product of its roots. If $P(0) = 4$, compute b.

I-5 $ABCDEFG$ is a pyramid with a hexagonal base. Compute the number of distinct ways all seven vertices of $ABCDEFG$ can be colored one of either red, blue, or green such that no vertices that share an edge are identically colored.

I-6 In triangle ABC, if $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, compute the smallest possible value of $\cos C$.

I-7 Compute $\log_2(3136) - \log_2(1764) + \log_2(900) - \log_2(400) + \log_2(144) - \log_2(36)$.

I-8 The roots of $10x^2 - 14x + k$ are $\sin\alpha$ and $\cos\alpha$ for some real value of α. Compute k.

I-9 Compute the rightmost non-zero digit in the base-8 expansion of $17!$.

I-10 Let S be a set of 100 points inside a square of side length 1. An ordered pair of not necessarily distinct points (P,Q) is *bad* if $P \in S$, $Q \in S$ and $|PQ| < \frac{\sqrt{3}}{2}$. Compute the minimum possible number of bad ordered pairs in S.

2017 ARML Local Individual Questions 1 and 2
(10 minutes)

Name: ______________________________

Team: ______________________________

Answer to I-1: [ ] Answer to I-2: [ ]

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2017 ARML Local Individual Questions 1 and 2
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I-1. A positive integer m is *stable* if $m = 2^n - n^2$ for some positive integer n. Compute the number of stable positive integers less than 2017.

I-2. Compute the area of the quadrilateral with vertices at $(1, 1)$, $(4, 7)$, $(5, 3)$, and $(2, 0)$.

2017 ARML Local Individual Questions 1 and 2
(10 minutes)

Name: ______________________________

Team: ______________________________

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2017 ARML Local Individual Questions 1 and 2
(10 minutes)

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2017 ARML Local Individual Questions 1 and 2
(10 minutes)

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2017 ARML Local Individual Questions 3 and 4
(10 minutes)

Name: ______________________________

Team: ______________________________

Answer to I-3: [ ] Answer to I-4: [ ]

I-3. The sum of the squares of five consecutive positive integers, the largest integer being n, is equal to the sum of the squares of the next four consecutive integers. Compute n.

I-4. The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the sum of its coefficients is equal to the sum of its roots and is also equal to the product of its roots. If $P(0) = 4$, compute b.

2017 ARML Local Individual Questions 3 and 4
(10 minutes)

Name: ______________________________

Team: ______________________________

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2017 ARML Local Individual Questions 3 and 4
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2017 ARML Local Individual Questions 3 and 4
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2017 ARML Local Individual Questions 5 and 6
(10 minutes)

Name: ______________________________

Team: ______________________________

Answer to I-5: [ ] Answer to I-6: [ ]

I-5. $ABCDEFG$ is a pyramid with a hexagonal base. Compute the number of distinct ways all seven vertices of $ABCDEFG$ can be colored one of either red, blue, or green such that no vertices that share an edge are identically colored.

I-6. In triangle ABC, if $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, compute the smallest possible value of $\cos C$.

2017 ARML Local Individual Questions 5 and 6
(10 minutes)

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Team: ______________________________

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2017 ARML Local Individual Questions 7 and 8
(10 minutes)

Name: ______________________________

Team: ______________________________

Answer to I-7: [ ] Answer to I-8: [ ]

I-7. Compute $\log_2(3136) - \log_2(1764) + \log_2(900) - \log_2(400) + \log_2(144) - \log_2(36)$.

I-8. The roots of $10x^2 - 14x + k$ are $\sin\alpha$ and $\cos\alpha$ for some real value of α. Compute k.

2017 ARML Local Individual Questions 7 and 8
(10 minutes)

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Team: ______________________________

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2017 ARML Local Individual Questions 9 and 10
(10 minutes)

Name: ______________________________

Team: ______________________________

Answer to I-9: [ ]   Answer to I-10: [ ]

I-9. Compute the rightmost non-zero digit in the base-8 expansion of 17!.

I-10. Let S be a set of 100 points inside a square of side length 1. An ordered pair of not necessarily distinct points (P, Q) is *bad* if $P \in S$, $Q \in S$ and $|PQ| < \frac{\sqrt{3}}{2}$. Compute the minimum possible number of bad ordered pairs in S.fa

2017 ARML Local Individual Questions 9 and 10
(10 minutes)

Name: ______________________________

Team: ______________________________

Answer to I-9: | | Answer to I-10: | |

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Team: ______________________________

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2017 ARML Local Relay 1
(6 minutes)

R1-1 If x and y are non-negative integers such that $x < y$ and $x!y! = 10!$, compute the maximum possible value of x.

2017 ARML Local Relay 1
(6 minutes)

R1-2 Let T = TNYWR. $HAIRNETS$ is an equilateral concave octagon of side length T. The interior angle measurements of H, I, N, and T are 60 degrees, and the interior angle measurements of A, R, E, and S are 210 degrees. Compute the area of $HAIRNETS$.

2017 ARML Local Relay 1
(6 minutes)

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2017 ARML Local Relay 2

(8 minutes)

R2-1 The side lengths of a triangle are $\sqrt{20}$, $\sqrt{17}$, and x. Compute the greatest possible integer value of x.

2017 ARML Local Relay 2
(8 minutes)

R2-2 Let $T = \text{TNYWR}$. The finite set S has exactly T distinct subsets each containing an even number of elements. Compute the number of elements in S.

2017 ARML Local Relay 2
(8 minutes)

R2-3 Let $T = \text{TNYWR}$. Let S consist of the set of non-negative integers less than T^3. Compute the (base-10) sum of the digits of all of the elements of S written in base T.

2017 ARML Local Relay 2
(8 minutes)

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2017 ARML Local Relay 3
(10 minutes)

R3-1 Compute the number of distinct ways to erase two of the decimal digits of 9876543210 and obtain an eight-digit number that is divisible by 9.

2017 ARML Local Relay 3
(10 minutes)

R3-2 Let T = TNYWR. ABC is an isosceles right triangle. If the longest median has length T, compute the area of ABC.

2017 ARML Local Relay 3
(10 minutes)

R3-3 Let T = TNYWR. The polynomial $x^3 + 3x^2 + px + T$ is evenly divisible by $x + 2$. Compute p.

2017 ARML Local Relay 3
(10 minutes)

R3-4 Let T = TNYWR. If p and q are the roots of $x^2 + x + 3$, then $x^2 + bx + c$ has roots $p + T$ and $q + T$. Compute c.

2017 ARML Local Relay 3
(10 minutes)

R3-5 Let T = TNYWR. Phil has a stack of money consisting of 5-dollar and 20-dollar bills worth 780 dollars in total. If there are T bills in total, compute the number of 5-dollar bills.

2017 ARML Local Relay 3
(10 minutes)

R3-6 Let $T = \text{TNYWR}$. Compute the remainder when the eight-digit number $\underline{T}\ \underline{2}\ \underline{3}\ \underline{4}\ \underline{2}\ \underline{3}\ \underline{4}\ \underline{T}$ is divided by 13.

2017 ARML Local Competition

Relay Round Answer Sheets

Team Name: **Relay 1, Team 1 Answer (3 minutes, 2 points)**	**Team Name:** **Relay 1, Team 1 Answer (6 minutes, 1 point)**
Team Name: **Relay 1, Team 2 Answer (3 minutes, 2 points)**	**Team Name:** **Relay 1, Team 2 Answer (6 minutes, 1 point)**
Team Name: **Relay 1, Team 3 Answer (3 minutes, 2 points)**	**Team Name:** **Relay 1, Team 3 Answer (6 minutes, 1 point)**
Team Name: **Relay 2, Team 1 Answer (4 minutes, 4 points)**	**Team Name:** **Relay 2, Team 1 Answer (8 minutes, 2 points)**
Team Name: **Relay 2, Team 2 Answer (4 minutes, 4 points)**	**Team Name:** **Relay 2, Team 2 Answer (8 minutes, 2 points)**
Team Name: **Relay 3, Team Answer (5 minutes, 6 points)**	**Team Name:** **Relay 3, Team Answer (10 minutes, 3 points)**

Name: ______________________________

Team: ______________________________

Time to submit answer (seconds): ______________________________

Answer to Tiebreaker: [ ]

2017 ARML Local Tiebreaker
(10 minutes)

Tiebreaker: $ABCD$ is an isosceles trapezoid with $\overline{AD} \parallel \overline{BC}$ and $AD < BC$. E lies on $\overline{BC}$ such that $\overline{AE} \perp \overline{BC}$ and let M be the midpoint of $\overline{BC}$. Lines $\overleftrightarrow{DE}$ and $\overleftrightarrow{AM}$ meet at G. Given that triangle GEM has area 20 and $AB = AM = 17$, compute the area of triangle ABC.

Name: ______________________________

Team: ______________________________

Time to submit answer (seconds): ______________________________

Answer to Tiebreaker: [ ]

2017 ARML Local Tiebreaker
(10 minutes)

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Team: ______________________________

Time to submit answer (seconds): ______________________________

Answer to Tiebreaker: [ ]

2017 ARML Local Tiebreaker
(10 minutes)

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Team: ____________________________

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Answer to Tiebreaker: | |
|---|

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Team: ________________________________

Time to submit answer (seconds): ________________________________

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2017 ARML Local Tiebreaker
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2017 ARML Local Competition

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Andy Niedermaier

If there are any questions about the contest, please contact the ARML Local Head Coordinator ASAP.

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